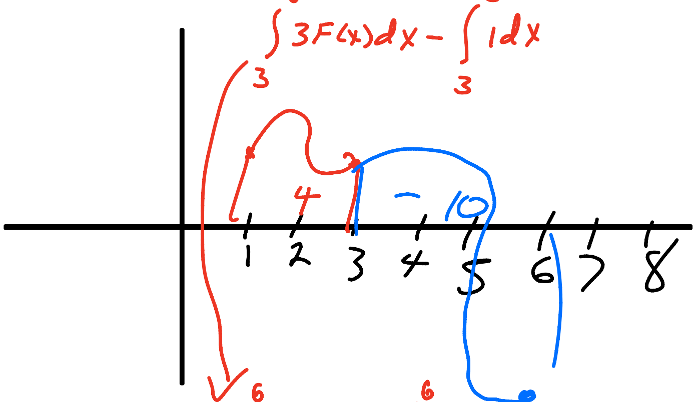
Period 3, Jan 29, 2025

$\int_3^6 1\,dx = x + C \Big[_3^6 = [6 + C] - [3 + C]$

$6 + C - 3 - C$

3. If $\int_1^3 f(x)\,dx = 4$ and $\int_1^6 f(x)\,dx = -6$, then $\int_3^6 [3f(x) - 1]\,dx =$ $\underline{-33}$
Show the set-up that led to your answer.

$\int_3^6 3F(x)dx - \int_3^6 1dx$



$3\int_3^6 F(x)dx - \int_3^6 dx$

$3(-10) - 3 = -30 - 3$

$\int_1^3 F(x)dx + \int_3^6 F(x)dx = \int_1^6 F(x)dx$

$4 + \int_3^6 F(x)dx = -6$

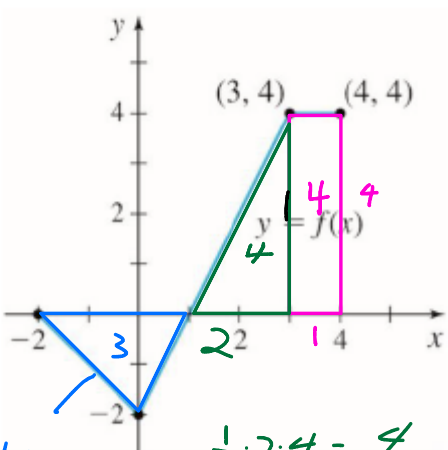
$-4 \qquad -4$

$\int_3^6 F(x)dx = -10$

$\int_3^6 1dx = x + C \Big[_3^6$

$6 - 3 = 3$

4. The graph of the piecewise function is below. What is $\int_{-2}^4 f(x)\,dx$?



$\frac{1}{2} \cdot 3 \cdot 2$

3

$\frac{1}{2} \cdot 2 \cdot 4 = 4$

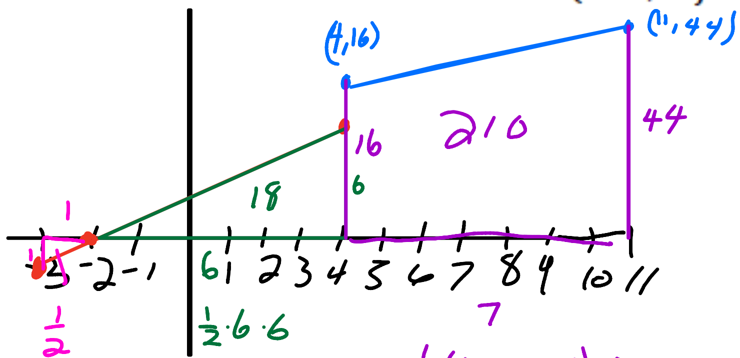
Left To Right

$\int_{-2}^4 F(x)dx = -3 + 4 + 4 = 5$

Right To Left

$\int_4^{-2} F(x)dx = -4 + -4 + 3$

5. Find $\int_{-3}^{11} h(x)\,dx$ where $h(x) = \begin{cases} x+2, & if\ -3 \le x \le 4 \\ 4x, & if\ 4 < x \le 11 \end{cases}$



$y = x+2$

x	y
-2	0
-3	-1
4	6

$y = 4x$

x	y
4	16
11	44

$\frac{1}{2}(16+44)\cdot 7$

$\frac{1}{2}\cdot 60 \cdot 7$

$30\cdot 7 = 210$

$$\int_{-3}^{11} h(x)\,dx = -\frac{1}{2} + 18 + 210 = 227\frac{1}{2}$$

$$\int_{-3}^{11} h(x)\,dx = \int_{-3}^{4} h(x)\,dx + \int_{4}^{11} h(x)\,dx = \int_{-3}^{4} (x+2)\,dx + \int_{4}^{11} 4x\,dx$$

$\frac{1}{2}x^2 + 2x + c \Big[_{-3}^{4}$

$\frac{1}{2}(4)^2 + 2(4) - \left[\frac{1}{2}(-3)^2 + 2(-3)\right]$

$8 + 8 - \frac{9}{2} + 6$

$17\frac{1}{2}$

$2x^2 + c \Big[_{4}^{11}$

$2(11)^2 - 2(4)^2$

$2(121) - 32$

$242 - 32 = 210$

$17\frac{1}{2} + 210 = 227\frac{1}{2}$

$\frac{1}{2}x^2+2x \Big[_{-2}^{4}$

$\left[\frac{1}{2}(4)^2+2(4)\right]-\left[\frac{1}{2}(-2)^2+2(-2)\right]$

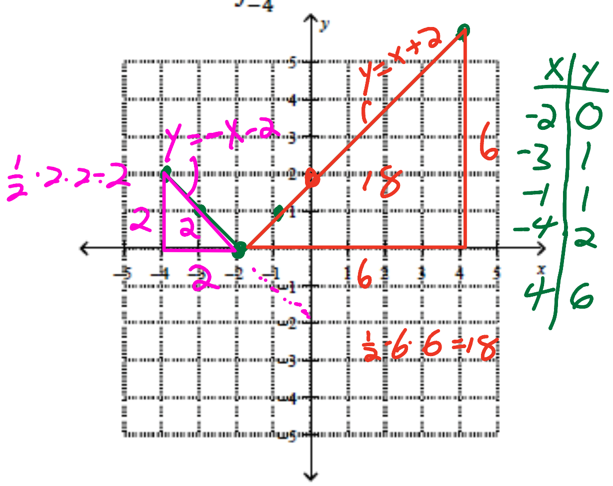
$8+8 - [2-4]$

$16 - -2 = 16+2 = 18$

$\int_{-4}^{4}|x+2|dx = \int_{-4}^{-2}(-x-2)dx + \int_{-2}^{4}(x+2)dx$

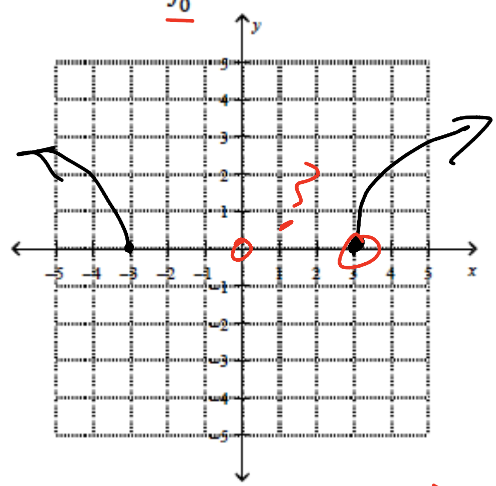
$-\frac{1}{2}x^2-2x \Big[_{-4}^{-2} + \frac{1}{2}x^2+2x \Big[_{-2}^{4}$

$\int_{-4}^{4}|x+2|\,dx =$ 2+18=20



2+18=20

$\int_{0}^{3}\sqrt{t^2-9}\,dt =$ ______



$-\frac{1}{2}x^2-2x \Big[_{-4}^{-2} = \left[-\frac{1}{2}(-2)^2-2(-2)\right]$

$-\left[-\frac{1}{2}(4)^2-2(-4)\right]$

$= [-2+4] - [~~-8+8~~]$

2

Example Set 1

1. Evaluate $\int_1^2 \left(x^2 - 3\right)dx$

2. Evaluate $\int_1^4 3\sqrt{x}\,dx$

$\int_1^4 3x^{\frac{1}{2}}dx = 3 \cdot \frac{2}{3} \cdot x^{\frac{1}{2}+1=\frac{3}{2}} + c \Big|_1^4$

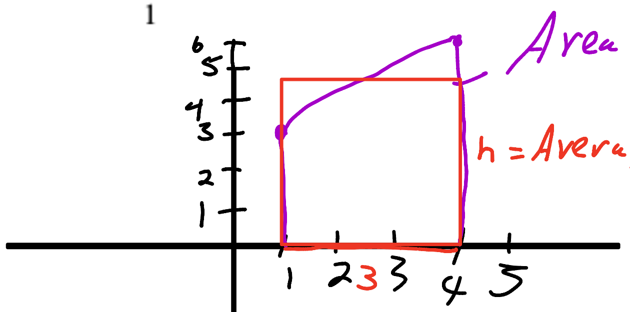
$= 2 \cdot x \cdot \sqrt{x} + c \Big|_1^4$

$2 \cdot 4 \cdot \sqrt{4} - 2 \cdot 1 \cdot \sqrt{1}$

$2 \cdot 4 \cdot 2 - 2 \cdot 1 \cdot 1$

$16 - 2 = 14$

2. Evaluate $\int_1^4 3\sqrt{x}\,dx$



Area = 14

h = Average value

$\frac{3 \cdot h}{3} = \frac{14}{3}$

$h = \frac{14}{3} = \text{Average value}$

$\int_1^4 3\sqrt{x}\,dx = (4-1)\begin{pmatrix}\text{Average} \\ \text{Value}\end{pmatrix}$ $4-1=3$

$\frac{1}{4-1}\int_1^4 3\sqrt{x}\,dx = \text{Average Value}$

$\frac{1}{b-a}\int_a^b F(x)\,dx = \text{Average value of } F(x) \text{ From a To b}$

MVT

$F(c) = $ Average Value

where $a \le c < b$

$12^2 = 144$

$+25 = 12 + 13$

$13^2 = 169$

$+27 = 13 + 14$

$14^2 = 196$

$$\int_1^4 3\sqrt{x}\,dx = 14$$

$$\frac{1}{4-1} \cdot \int_1^4 3\sqrt{x}\,dx = \frac{14}{3} = \text{Average Value}$$

To Find C

$$\frac{14}{3 \cdot 3} = \frac{3\sqrt{c}}{3}$$

$$\left(\frac{14}{9}\right)^2 = (\sqrt{c})^2$$

$25^2 = 625$

$+26 + 25$

$26^2 = 676$

$+26 + 27$

$27^2 = 729$

Example 3: Find the **area** of the region bounded by the graph of $y = x^2 - 2x + 3$, the **x-axis**, and the **vertical lines $x = 0$** and **$x = 3$**.

$$Area = \int_0^3 (x^2 - 2x + 3)\,dx$$

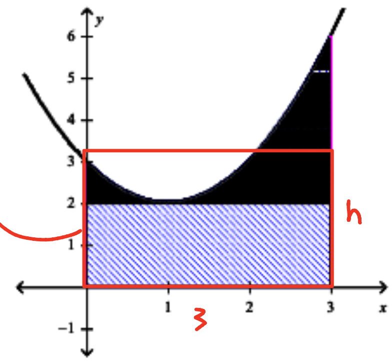
$$= \left(\frac{x^3}{3} - x^2 + 3x\right)\Bigg|_0^3$$

$$= \left(\frac{3^3}{3} - 3^2 + 3(3)\right) - (0)$$

$$= (3^2 - 3^2 + 3(3))$$

$$= 9$$

Area = 9



$3 \cdot h = 9$

$h = 3$

Average Value = 3

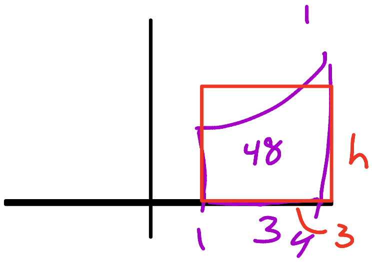
Example 7: Find the average value of $f(x) = 3x^2 - 2x$ on the interval [1, 4].

$$\int_1^4 (3x^2 - 2x)\,dx = 3 \cdot \frac{1}{3} \cdot x^{2+1=3} - 2 \cdot \frac{1}{2} \cdot x^{1+1=2} + C \Big[_1^4$$

$$x^3 - x^2 \Big[_1^4 = [4^3 - 4^2] - [1^3 - 1^2]$$

$$64 - 16 - 0$$

$$48$$



$3 \cdot h = 48$

$h = 16$

Average velocity from a to b

position function

$$\frac{1}{b-a}\int_a^b v(T)\,dT = \frac{S(b)-S(a)}{b-a} = \text{Average velocity}$$

$$\frac{\int_a^b v(T)\,dT}{b-a} = \frac{\text{distance Traveled}}{\text{Time}} = \text{Average velocity}$$

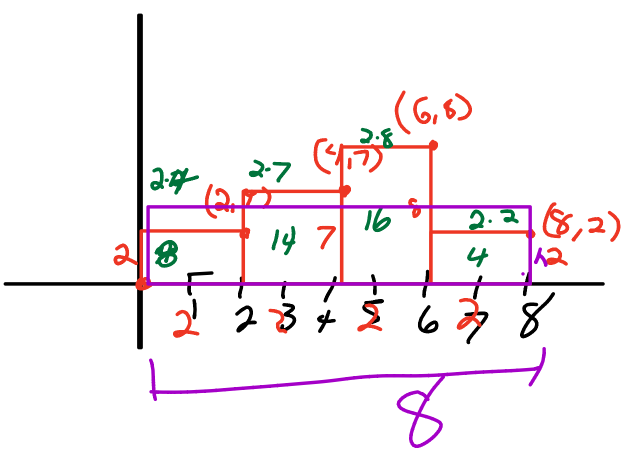
Average acc from a to b

$$\frac{1}{b-a}\int_a^b a(T)\,dT = \frac{V(b)-V(a)}{b-a} = \frac{\text{change in velocity}}{\text{Time}} = \text{Average acceleration}$$

A car's acceleration a in ft/s^2 is measured each second t for $t = 0$ to $t = 8$ and posted in the table.

t	0	1	2	3	4	5	6	7	8
$a(t)$	0	2	4	6	7	7	8	6	2

Use a Right Riemann sum with 4 subintervals of equal length to approximate the car's average velocity over the interval from 0 to 8 seconds.



$8 + 14 + 16 + 4 = 42$

$22 + 20 = 42$

$8 \cdot h = 42$

$h = \frac{42}{8} = \frac{21}{4} = 5\frac{1}{4}$

Average velocity